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Chain rule problems pdf

When this message is displayed, it means that we are having trouble loading external resources on our website. If you are behind a web filter, please make sure that the *.kastatic.org and *.kasandbox.org domains are unmarked. Built with the help of Alexa Bosse. Chain Rule: Problems and SolutionsDo you work on calculating derivatives using the chain rule in Calculus? Let's solve some common problems step by step so that you can learn to solve them routinely for yourself. Do you want to check the calculation of derivatives that do not require the chain rule? This material is there. Do you want to skip the summary? Jump on problems and their solutions. CALCULUS SUMMARY: Chain rule You can access our mobile phone table of derivatives and differentiation rules at any time via the menu item Key Formulas at the top of each page. Click Hide Summary/Show You must use the Chain Rule to find the derivation of a function that consists of a function within another function. For example, the inner function consists $e $e $x , () () Because each of these functions consists of a function within another function $t called a composite function, we must use the chain rule to find its derivation, as shown in the following problems. How can I tell what the internal and external functions are? Here's a foolproof method: Imagine calculating the value of the function for a specific value of $x and identify the steps you would take, because you always automatically start with the inner function and work your way into the outer function. For example, imagine calculating the calculation of the left (x-2+1-right) for $x=3. Without thinking about it, first calculate $x 2 + 1 (which corresponds to 3,2 +1 = 10), so this is the inner function guaranteed. Then you would next calculate at 10.7, and so is the outer function (this imaginary computing process works each time to correctly identify what the internal and external functions are. Chain RuleView a composite function whose outer function is $f(x) and whose inner function is $g(x). The composite function is therefore $f(g(x)). f'''Big(g(x)'Big) 'cdot g'(x) '[5px]&='text'[derivation of the outer function], evaluated at the inner function], If we write $y = f(u) and $u = g(x), then ['bbox[yellow,8px]'dfrac'dy'dx' = 'dfrac'dy'du'informal: Although few people admit it, almost everyone in the blue boxes above thinks about the informal approach. We will illustrate the following problems. Chain rule example #1 differentiate $f(x) = (x-2 + 1).7. Solutions. We solve this with three different approaches – but we recommend that you familiarize yourself with the third approach as soon as possible, as you will use this to calculate derivatives quickly, as the price progresses.• Solution 1. Let's use the first form of the chain rule above: '['bbox[10px,border:2px dashed blue]'begin'align*'left[f'Big (g(g(x)'Big)'right]' &= f''Big(g(x)'Big) 'cdot g'(x) '[5px]&='text'[the rated at the inner function] . $f . inner function $u = g(x) = x2 +1. Then $f'(u) = 7u,6, and $g'(x) = 2x. We could, of course, simplify the result algebraically to 14x(x-2+1), but we leave the result as it is written to emphasize the chain rule term 2x at the end.• Solution 2. Let's use the second form of the chain rule above: '['bbox[10px,border:2px dashed blue]'dfrac'dy'dy'dx' = 'dfrac'dy'du' 'cdot 'dfrac'du'dx'' we have $y = u'7' and $u = x'2 +1. you = 7u,6, and dfrac and dfrac =dx. Therefore, begin-Direction3-, dfrac-Dy-Dx-) &= 7u-6-Cdot 2x-[8px] &= 7(x-2+1)-6-Cdot 2x-quad- With some experience you will not introduce a new variable like $u = cdots, as we did above. Instead, you think something like: The function is a bunch of things to the 7th Power. So the derivation is 7 times the same stuff to the 6th power, sometimes the derivation of this stuff. with the same stuff inside from the right] ,) X-text,text and text- and -/[12px]-text,-text,-phantom,-F(x)= , [8px] &= 7(x-2 + 1)-6 -Cdot (2x) ,quad-cmark-end-align*- Note: You would never really stuff = ... To write. Instead, just keep in mind what this substance is and continue to write down the required derivatives. There are many fully solved sample problems below! [Collapse] Chain Rule & Power Rule 'begin'align*'You usually see that this is written as dfrac (d. Dx) (u,0, right) = n u'n-1', 'cdot'dfrac'd'dx' illustrate the following five problems. Chain rule problem #1 differentiate $f(x) = left (3x2 - 4x + 5 x right) in these two ways. The first is the way most experienced people quickly develop the answer, and that we hope that you will soon be satisfied with them. The second is more formal. Solution 1 (fast, the way most people think). Think of something like: The feature is a pair of stuff to the eighth power. So the derivation is eight times as much as the seventh power, sometimes the derivation of this stuff. with the same stuff inside from the right] ,) »quad 'text''stuff' = 3x'2 – 4x + 5 »[12px] 'Text',Dann''phantom'f'f'dfrac'df'df'dx''dx'dx'2 – 4x + 5-right) ,[8px] &= 8'left(3x'2 – 4x + 5'right)'7 'cdot (6x -4)'quad 'cmark'[8px] 'end'align*' Note: You would never really stuff = ... Instead, just keep in mind what this substance is and continue to write down the required derivatives. Solution 2 (more formal). Let's use the first form of the chain rule above: '['bbox[10px,border:2px dashed blue]'begin'align*'left[f'Big(g(x)'Big)'right]' &= f'Big(g(x) 'Big) 'cdot g'(x) '[5px]&='text'[the We have the outer function $f(u) = u'8' and the inner function $u = g(x) = 3x-2 - 4x + 5.' Then $f'' (u) = 8u'7,' and $g'(x) = 6x -4 Begin-&-F'(x) &= 8u-7-Cdot (6x - 4) &= 8,8 , Links (3x- + 5- right) and Cdot (6x-4) Quad (End Alignment)[8px] &= 8,8 , Left $f #2 (3x - 5 x 8) : Remember also the Dfrac-Dx-Tan-X-Tan-Tan-No. X - show big solution. The first is the way most experienced people quickly develop the answer, and that we hope that you will soon be satisfied with them. The second is more formal. Solution 1 (fast, the way most people think). Think of something like: The feature is a pair of stuff to the power of 3. So the derivation is three times as much as the 2,- and-times the derivation of this stuff. with the same stuff inside from the right] ,) »quad 'text'stuff' = 'tan x'[12px] 'text'Then'phantom'f''f'8px] &= 3'big['tan x'big]'2 'cdot 'sek'2 x '[8px] &= 3'tan'2 x 'cdot'2 x 'quad 'cmark'[8px] 'end'align*' Note: You would never really stuff = ... Instead, just keep in mind what this substance is and continue to write down the required derivatives. Solution 2 (more formal) . Let's use the first form of the chain rule above: . ['bbox[10px,border:2px dashed blue]'begin'align*'left[&= f'''Big(g(x)'Big) 'cdot g'(x) '[5px]&='text'[Derivation of the outer function, We have the outer function $f(u) = u'3' and the inner function $u = g(x) = tan x. Then $f'(u) = 3u,2, and $g'(x) = ssec.2 x.' (Remember, that (tan x) = Therefore beginalign* f'(x) &= 3u'2 'cdot ('sek'2 x)'[8px] &= 3'big['tan x'big]'2 'cdot'2 x '[8px] &= 3't an''&=3'tan'' 2 x 'cdot 'sek'2 x 'quad'cmark'[8px] 'end'align*' [collapse] Chain Rule Problem #3 Differentiate $f(x) = ('cos x - 'sin x)'-2'.' Click on Calculus Solution Ads We will solve these two ways again. The first is the way most experienced people quickly develop the answer, and that we hope that you will soon be satisfied with them. The second is more formal. Solution 1 (fast, the way most people think). Think of something like: The feature is a pair of things to the power of '-2'. So the derivation is two times the same stuff that is the power of 3 times the derivation of this stuff. with the same stuff inside from the right] ,) quad (text) = cos x x - sin x[12px] Text Then Phantom (x)= dfrac dfrac df dx &= -2(text)-3 cdot dfracddx (cos x [8px] &= -2(-cos x - sin x) - cdot (--sin x - écos x)-quad-cmark-[8px] -End-Align*. We could simplify the answer by calculating the negative characters from the last term, but we prefer to stop there to keep the focus on the chain rule. Note: You would never really stuff = ... Instead, simply keep in mind what this substance is and continue to write down the required derivatives. Solution 2 (more formal). Let's use the first form of the chain rule above: '['bbox[10px,border:2px dashed blue]'begin'align*'left[f'Big(g(x)'Big)'right]' &= f''Big(g(x)'Big) 'cdot g '(x) '[5px]&='text'[the We have the outer function $f(u) = u'-2' and the inner function $u = g(x then $f'(u) = -2u-3, and $g'(x) = -sin x – écos x. (Remember, that (cos x) = -' sin x, and sin x) = cos x) Therefore it is no longer so, that begin/align* f'(x) &= -2u--3- ,cdot (-sin x- 'cos x) '[8px'] &= -2(-cos x – sin x) . , but we prefer to stop there to keep the focus on the chain rule. [Collapse] Chain Rule Problem #4 differentiate $f(x) = 'left(x'5 + e'x'right)'{99}.' Click here to view Calculation Solution 1 (fast, as most people think). Think of something like: The feature is a pair of stuff to the 99th Power. The derivative is therefore 99 times as much as the 98. of this stuff. with the same stuff inside from the right] ,{99},) quad (text) = x-5 + x-[12px text[12px] text phantom (x)= dfrac dfrac dx &= text (text) {98} cdot dfrac ddx (x-5) + ex ,[8px] &= 99,links) (x5 + ex) {98} cdot (5x x x) , quad[8px] endalign* Note: Stuff = ... Instead, just keep in mind what this substance is and continue to write down the required derivatives. Solution 2 (more formal). Let's use the first shape of the chain rule above: '['bbox[10px,border:2px dashed blue]'begin'align*'left[f'Big(g(x)'Big)'right]' &= f'Big(g(g(x 'cdot g'(x) '[5px]&='text'[the We have the outer function $f(u) = u-{99} and the inner function $u = g(x) = x Then $f'(u) = 99u,{98}, and $g'(x) = 5x'4 + e'x.' Therefore 'begin'align*' f'(x) &= 99u'{98} 'cdot (5x'4 + e'x) &= 99 {98} ,cdot (5x) (5x) ,-$f #5, think of something like: the function is 'sqrt'text'stuff'. So the derivation is dfrac{1}{2}dfrac{1},sqrt, text, the same stuff, sometimes the derivation of this stuff. (Open to develop these derivatives.) Keep in mind that the rule also applies to fracture forces: {1}{2}, frac{1}{2}u,links(frac{1}{2} – 1 ,,right) &= frac{1}{2}u-1/2-[8px]&=,- frac{1}{2} frac{1} sqrt end-Align* *- dfrac-d-dx-links(x-2 + 1 ,2) &= dfracddx-links(x-2-right) + {0}dpx,border:2px dashed blue ('dfrac'df'dx'' = 'left'dfrac'df'df'd'text''text','text', with the same stuff inside'' 'right] 'mal'dfrac'd'text' 'quad 'text'stuff' = x'2 + 1'[12px] 'text'Then'phantom'f''f'{1}{1}{2}'&= frac{1}{2} ,frac{1}sqrt (2x) (2x) Quad (2x) and Solution 2. Let's use the first form of the chain rule above: '['bbox[10px,border:2px dashed blue]'begin'align*'left[f'Big(g(x)'Big)'right]' &= f''Big(g(x)'Big) 'cdot g'(x) '[5px]&='text'[der The inner function] , [5px]&,qquad,-times,text [derivation of the inner function], end, alignment* ,] We have the outer function $f(u) = sqrt- and the inner function $u = g(x) = x-2 + 1. Then left ('sqrt'u'' ' dfrac{1}{2},dfrac{1}, sqrt and left (x 2 + 1 ,, right{1}{1}{2}{1}{1}{2}. sqrt x dx &= e-text(the stuff) , cdot 'dfrac'd'dx'dx'('text', the stuff') 'end'align*' You will usually see that this is written as 'dfrac'd'dx'e'u = e'u 'cdot'. Chain Rule Problem #6 differentiate $f(x) = e'sin x'. Click here to view Calculation Solution 1 (fast, as most people think). Think of something like this: the function is $e to the power of some things. So the derivation is $e to the power of exactly the same stuff, sometimes the derivation of this stuff. with the same stuff inside from the right] ,/,-,) »quad 'text'stuff' = 'sin x'[12px] 'text'Then'phantom'f''f'd-dx-Text-(substance)-(substance)-,[8px] &=e-Sin x- x -Quad-Cmark-[8px] Instead, just keep in mind what this substance is and continue to write down the required derivatives. Solution 2 (more formal). Let's use the first shape of the chain rule above: '['bbox[10px,border:2px dashed blue]'begin'align*'left[f'Big(g(x)'Big)'right]' &= f ''Big(g(x)'Big) 'cdot g'(x) '[5px]&='text'[we have the outer function $f(u) = e'u' and the inner function $u = g(x) = 'sin x.' Then $f'(u) = e'u,' and $g''* f'(x) &= e'u 'cdot 'cos x '[8px] &= e'sin x' 'cdot 'cos x 'cmark 'end'align*' [collapse] Chain Rule Problem #7 Differentiate $f(x) = e'left(x'7 – 4x'3 + x 'right)' , the way as most people think). Think of something like this: the function is $e to the power of some things. So the derivation is $e to the power of exactly the same stuff, sometimes the derivation of this stuff. with the same stuff inside from the right] ,/,-,) X-Text-, Text = x-7 - 4x-3 + x -[12px] ,Text-Text,-Phantom-F(x)= dfrac dfrac df dx &= left (e-text,text () – 4x 3 + x right) , cdot ,,links(7x-6 – 12x-2 + 1 ,, right) , quad ,[8px] , End-Align*- Note: You would never really stuff = ... Instead, just keep in mind what this substance is and continue to write down the required derivatives. Solution 2 (more formal). Verwenden wir die erste Form der Chain-Regel oben: '['bbox[10px,border:2px gestrichelt blue]'begin'align*'left[f'Big(g(x)'Big)'right]' &= f''Big(g(x)'Big) 'cdot g'(x) '[5px]&='text'[der Wir haben die äußere Funktion $f(u) = e'u' und die innere Funktion $u = dann $f'(u) = e'u, und $g'(x) = 7x-6 -12x-2 +1.' '(x) &= e'u 'cdot 'left(7x'6 -12x'2 +1 'rechts''[8px] &= e'sin x' 'cdot 'left(7x'6 -1 2x 2 +1 ,,rechts) ,,quadcmark endalign* [Reduzieren] Kettenregel & Trig-Funktionen , begin, align*, Text und Wenn && f(x) &= sintext(stuff) , \\[8px] \text{then} &&\dfrac{df}{dx} &= \cos\text{(that stuff)}\cdot \dfrac{d}{dx}(\text{that stuff}) \end{align*} You'll usually see this written as $$\dfrac{d}{dx}\sin u = \cos u \cdot \dfrac{du}{dx}$$ $$ — $$ \begin{align*} \text{If} && f(x) &= \cos\text{(stuff)} , [8px] .-----------------'dfrac'd'dx'cos u = -'sin u'cdot'dfrac'du'dx'dx'[8px] .[8px] .[8px] .[8px) .[8px] .[8px) .[8px] .[8px) .[8px .[8px .[8px] .[8px] .[8px] .[8px .[8px] .-You'll usually see this written as 'dfrac'd'dx'tan u = 'sec'2 u'cdot 'dfrac'du'dx' The next two problems illustrate. Chain rule problem #8 differentiate $f(x) = sin(2x). Think of something like: Function is sin (of some things). So the derivation is cos (of the same stuff), sometimes the derivation of this stuff. with the same stuff inside from the right] ,/,-,-,), 'quad 'text'stuff' = 2x '[12px] 'text'Then'phantom'f'f'f'/x)= 'dfrac'df'dx'df'df'dx'dfrac-d-dx-(-text-stuff)-right)[4px &= cos(2x) cdot(2) quad 10px,border:2px dashed blue]'begin'align*'left[f'Big(g(x)'Big)'right]' &= f''Big(g(x)'Big) 'cdot g'(x) '[5px]&='text'[the we have the outer function $f(u) = sin u and the inner function $u = g(x) $g $f = 2 ,cmark $f #9 Think of something like: The function is tan(of some things). So the derivation is sec.2 (of the same stuff) and the time the derivation of this stuff. [,'with the same stuff inside from the right] ,/,) text text = e-x-[12px] text, Then ,phantom/x)= dfrac dfrac df dx &= sec.2)('dfrac'd'dx'text'') ' Let's use the first form of the chain rule above: '['bbox[10px,border]dashed blue]'begin'align*'left[f'Big(g(x)'Big)'right]' &= f''Big(g(x)'Big) 'cdot g'(x) '[5px]&='text'[the We have the outer function $f(u) = tan u and the inner function $u = g(x) = ex $g $f. (x) = ex. , or require the chain rule to be used more than once. Chain rule problem #10 This problem combines the product rule with the chain rule. Distinguish $f(x) = 'left(x'2 + 1 'right)'7 (3x – 7)'4.' Click to view the calculation solution. We will not write fabric, as we did before to use the chain rule, and instead simply write down the answer with the same thinking as above: Left (left) & 1 ,,right) = 7'left(x'2 + 1 'right)'6 'cdot (2x) 'Opened for more details We can 'left(x'2 + 1 'right)'7' as '('text''' Dann wird begin-Aus/b/f(x) &= (Text- und Material-) \quad \text{stuff} = x^2 + 1 \\[12px] \text{Then}\\ f'(x) &= 7(\text{stuff})^6 \cdot (x^2 + 1)' \\[8px] &= 7(x^2 + 1)^6 \cdot (2x) \end{align*}[collapse] and $$\left[(3x – 7)^4 \right]' = 4(3x – 7)^3 \cdot (3) $$Now let's use the Product Rule: \[\begin{align*} (f g)' &= \qquad f' g\qquad\qquad +\qquad\qquad fg' \\[8px] \left[\left(x^2 + 1 \right)^7 (3x – 7)^4 \right]' &= \left[\left(x^2 + 1 \right)^7\right]' (3x – 7)^4\, + \,\left(x^2 + 1 \right)^7 \left[(3x – 7)^4 \right]' \\[8px] &= \left[7\left(x^2 + 1 \right)^6 \cdot (2x) \right](3x – 7)^4 + \left(x^2 + 1 \right)^7 \left[4(3x – 7)^3 \cdot (3) \right] \quad \cmark \end{align*} \] —– We could of course simplify this expression algebraically : $f'(x) = 14x-left (x-2 + 1 ,right)-6 (3x – 7)-4 + 12 'left(x'2 + 1 'right)'7 (3x – 7)'3 'Instead, we stopped where we were above to emphasize the way we developed the result, which is most important here. we don't need to show you how to make algebra!) [Collapse] Chain rule problem #11 This issue requires the double use of the chain rule. Distinguish $f(x) = écos('tan(3x)).' Click here to view Calculation Solution 1 (fast, the way most people think). Think of the approximate beautiful kind: the overall function is cos. the first part of the derivative is --sin(text of the exact same stuff $f s A). the first inner function is Tan(overbrace (text) of some different things B,3x), and since the derivation of Tan Sek. 2 is, the next part of the derivative is sek.2 (text of the exact same stuff B $f(x) = sin(tan(3x) cdot (3x) x () cdots and : $f'(x) = -'sin('tan(3x)) 'cdot'sek'2(3x) 'cdot 3 'quad 'cmark' And you're done! (Or as they say in some parts of the world: And Bob is your uncle!) Solution 2 (more formal). Although it is tedious to write every single function, we use an extension of the first form of the chain rule above, now applied to $f-Bigg(g-Big(h-big)-Big-)::[-bbox[10px,border:2px dashed blue]'begin'align*'left[f'Bigg(g'Big(h(h(x)'Big)'Big g) 'right]' &= f''Bigg(g'Big(g(x)'Big)'Big)'Big) 'cdot g'Big(h(x)'Big' 'cdot h'(x) '[5px]&='text'[derivation of external function, Evaluated at the middle function] $z $f . then $u $f'(z) = -sin z, and $g'(u) = ssec'2 u,' and $h'(x) = 3.' Therefore: 'begin'align*' f'(x) &= (-sin z) dot ('sek'2 u)'cdot ''#12 $f' Note: Remember that the Sin-9(cdots) = bigsin(cdots) big9. Click The Calculated Solution. and instead the way you deal with: The outermost function is , (overbrace('overbrace'text'some stuff''some stuff A'1 + sin'9(2x + 3))', and so is the first part of the derivative, 2.9 x (,text, the exact same stuff A) : begin'align*' f'(x) &= 2'left(1 + 'sin'9(2x + 3) 'times 'cdots' & 'phantom'= 2 Next we consider the first inner Function (the substance A was) : 1 + ('overbrace''text'some stuff'B'sin(2x + 3)''9.' So the next part of the derivative is '9'times ('text'that exact same stuff B')''8': f'(x) &= 2 'left(1 + 'sin'9(2x + 3) 'right) 'cdot 9'big['sin(2x+3)'big]'8''mal 'cdots'[8px] &= 2'left(1 + 'sin'9(2x + 3) 'right) 'cdot 9'sin'8(2x+3) 'times 'cdots'' & Phantom (2x) (1 + sin9(2x + 3) right) cdot 9'sin'8(2x+3) 'cdot'cos(2x+3)'cdot 2' ,- and so the next part of the derivative cos (text) is exactly the same stuff C) : beginalign* f'(x) &= 2'left(1 + sin'9(2x + 3) 'right) 'cdot 9'sin'8(2x+3) 'cdot'' cos(2x+3) - times 'cdots' & 'phantom'= 2'left(1 + 'sin'9(2x + 3) 'cdot 9'sin'8(2x+3) 'cdot 'cos(2x+3)'cdot 2' 'end'align*' Finally , the last inner function (that was Stuff C) is 2x +3, and so the last part of the derivative is 2,00 € (f'(x) = 2 'left(1 + 'sin'9(2x + 3) 'right) 'cdot 9'sin'8(2x+3) 'cdot 'cos(2x+3)'cdot 2 ''cmark' [Collapse] Must Do you use the derivation to find the equation of a tangent line (or the equation of a normal line)? We have a separate page on this subject here. 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